Derivatives & Antiderivatives

General Rules

Linearity	$(af)' = af'$ $(f+g)' = f'+g'$	$\int af = a\int f$ $\int (f+g) = \int f + \int g$	Linearity
Product Rule	$(fg)' = f'g + fg'$	$\int u\mathrm{d}v = uv - \int v\mathrm{d}u$	Int. by parts
Division Rule	$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$		
Chain Rule	$(f\circ g)' = (f'\circ g)g'$	$\int f(g(x))g'(x)\mathrm{d}x \stackrel{\left.\begin{array}{l}u=g(x)\\ \mathrm{d}u=g'(x)\mathrm{d}x\end{array}\right\}}{=} \int f(u)\mathrm{d}u$	u-substitution

Above: $a\in\mathbb{R}$ is a constant, and f and g functions.

Elementary Functions (Powers and Partial Fractions)

$f(x)$	$f'(x)$	$\int f(x)\mathrm{d}x$
a	0	$ax + C$
x	1	$\frac{x^2}{2} + C$
x^2	$2x$	$\frac{x^3}{3} + C$
x^α	$\alpha x^{\alpha-1}$	$\frac{x^{\alpha+1}}{\alpha+1} + C \ (\alpha \neq -1)$ $\ln\lvert x\rvert + C \ (\alpha = -1)$
$\frac{1}{x}$	$\frac{-1}{x^2}$	$\ln\lvert x\rvert + C$
$\frac{1}{x-\zeta}$	$\frac{-1}{(x-\zeta)^2}$	$\ln\lvert x-\zeta\rvert + C$
$\frac{1}{(x-\zeta)^k}$	$\frac{-k}{(x-\zeta)^{\alpha+1}}$	$-\frac{1}{k-1}\frac{1}{(x-\zeta)^{\alpha-1}} + C \ (k\geq 2)$
$\frac{1}{x^2+a^2}$	$\frac{-2x}{(x^2+a^2)^2}$	$\frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$
$\frac{1}{(x^2+a^2)^k}$	$\frac{-2kx}{(x^2+a^2)^{k+1}}$	$\frac{1}{2(k-1)a^2}\frac{x}{(x^2+a^2)^{k-1}} + \frac{2k-3}{2(k-1)a^2}\int\frac{\mathrm{d}x}{(x^2+a^2)^{k-1}} \ (k\geq 2)$
$\frac{x}{x^2+a^2}$	$\frac{2a^2}{(x^2+a^2)^2} - \frac{1}{x^2+a^2}$	$\frac{1}{2}\ln\lvert x^2+a^2\rvert + C$
$\frac{x}{(x^2+a^2)^k}$	$\frac{2ka^2}{(x^2+a)^{k+1}} - \frac{2k-1}{x^2+a^2}$	$-\frac{1}{2(k-1)}\frac{1}{(x^2+a^2)^{k-1}} + C \ (k\geq 2)$

Above: $c\in\mathbb{R}$, $\alpha,\zeta\in\mathbb{R}$, $k\in\mathbb{N}$ and $a>0$ are constants,
and C the antiderivative constant.

Elementary Functions (Exponentials and Logarithms)

$f(x)$	$f'(x)$	$\int f(x)\mathrm{d}x$
e^x	e^x	$e^x + C$
$\ln x$	$\frac{1}{x}$	$x\ln x - x + C$
b^x	$(\ln b)b^x$	$\frac{1}{\ln b}b^x + C$
$\log_b x$	$\frac{1}{(\ln b)x}$	$x\log_b x - \frac{x}{\ln b} + C$

Above: $b > 0$ are constants, and C the antiderivative constant.

Elementary Functions (Trigonometric Functions)

$f(x)$	$f'(x)$	$\int f(x)\mathrm{d}x$
$\sin x$	$\cos x$	$-\cos x + C$
$\cos x$	$-\sin x$	$\sin x + C$
$\tan x$	$\sec^2 x = 1 + \tan^2 x$	$\ln\lvert\sec x\rvert + C$
$\csc x$	$-\csc x \cot x$	$-\ln\lvert\csc x + \cot x\rvert + C$
$\sec x$	$\sec x \tan x$	$\ln\lvert\sec x + \tan x\rvert + C$
$\cot x$	$-\csc^2 x = -1 - \cot^2 x$	$-\ln\lvert\csc x\rvert + C$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$	$x\arcsin x + \sqrt{1-x^2} + C$
$\arccos x$	$\frac{-1}{\sqrt{1-x^2}}$	$x\arccos x - \sqrt{1-x^2} + C$
$\arctan x$	$\frac{1}{1+x^2}$	$x\arctan x - \frac{1}{2}\ln\lvert 1+x^2\rvert + C$
$\operatorname{arccsc} x$	$\frac{-1}{x\sqrt{x^2-1}}$	$x\operatorname{arccsc} x + \arcsin x + C$
$\operatorname{arcsec} x$	$\frac{1}{x\sqrt{x^2-1}}$	$x\operatorname{arcsec} x + \arccos x + C$
$\operatorname{arccot} x$	$\frac{-1}{1+x^2}$	$x\operatorname{arccot} x + \frac{1}{2}\ln\lvert 1+x^2\rvert + C$

Above: C is the antiderivative constant.

ELEMENTARY FUNCTIONS (Square Roots of Quadratic Polynomials)

$f(x)$	$f'(x)$	$\int f(x)\mathrm{d}x$
$\sqrt{a^2-x^2}$	$\frac{-x}{\sqrt{a^2-x^2}}$	$\frac{a^2}{2}\arcsin\frac{x}{a}+\frac{1}{2}x\sqrt{a^2-x^2}+C$
$\sqrt{x^2+a^2}$	$\frac{x}{\sqrt{x^2+a^2}}$	$\frac{1}{2}x\sqrt{x^2+a^2}+\frac{a^2}{2}\ln\left\lvert x+\sqrt{x^2+a^2}\right\rvert+C$
$\sqrt{x^2-a^2}$	$\frac{x}{\sqrt{x^2-a^2}}$	$\frac{1}{2}x\sqrt{x^2-a^2}-\frac{a^2}{2}\ln\left\lvert x+\sqrt{x^2-a^2}\right\rvert+C$
$\frac{1}{\sqrt{a^2-x^2}}$	$\frac{x}{(a^2-x^2)^{\frac{3}{2}}}$	$\arcsin\frac{x}{a}+C$
$\frac{1}{\sqrt{x^2+a^2}}$	$\frac{-x}{(x^2+a^2)^{\frac{3}{2}}}$	$\ln\left\lvert x+\sqrt{x^2+a^2}\right\rvert+C$
$\frac{1}{\sqrt{x^2-a^2}}$	$\frac{-x}{(x^2-a^2)^{\frac{3}{2}}}$	$\ln\left\lvert x+\sqrt{x^2-a^2}\right\rvert+C$

Above: $a>0$ is a constant, and C the antiderivative constant.