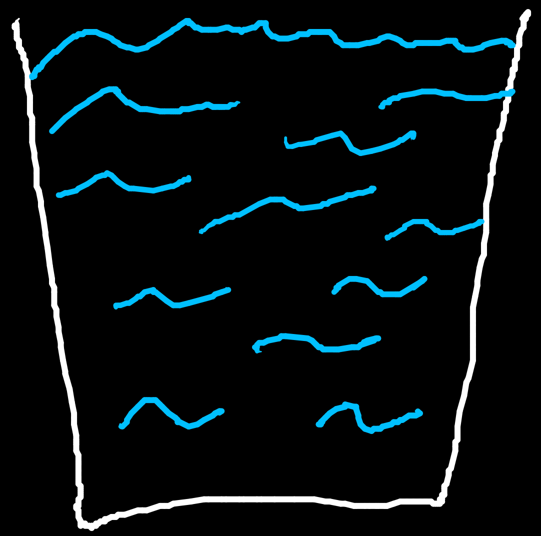
How CAREFUL do YOU have to be to SOLVE a System of Equations?

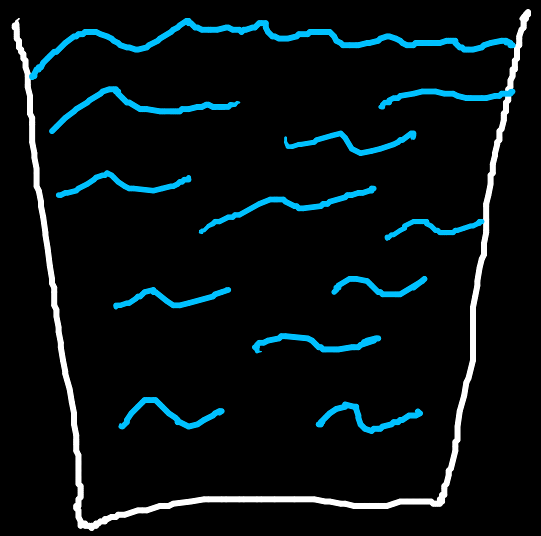
Josué Tonelli-Cueto

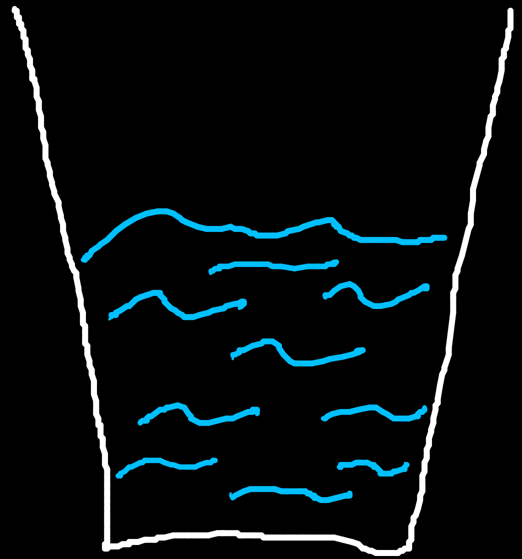
Moving a glass

Moving a glass



Moving a glass





ALL ERRORS
ARE DIFFERENT,

ALL ERRORS
ARE DIFFERENT,
BUT SOME ERRORS
ARE MORE DIFFERENT
THAN OTHERS

$$\begin{cases} 1.00\,x + 1.01\,y = 1.00 \\ 2.00\,x + 2.50\,y = 1.00 \end{cases}$$

$$\begin{cases} 1.00\,x + 1.01\,y = 1.00 \\ 2.00\,x + 2.50\,y = 1.00 \end{cases} \rightsquigarrow \begin{array}{l} x \approx 3.10\ldots \\ y \approx -2.08\ldots \end{array}$$

$$\begin{cases} 1.00\,x + 1.01\,y = 1.00 \\ 2.00\,x + 2.50\,y = 1.00 \end{cases} \rightsquigarrow \begin{matrix} x \approx 3.10\ldots \\ y \approx -2.08\ldots \end{matrix}$$

vs.

$$\begin{cases} 1.00\,x + 1.01\,y = 1.00 \\ 2.00\,x + 2.10\,y = 1.00 \end{cases}$$

$$\begin{cases} 1.00\,x + 1.01\,y = 1.00 \\ 2.00\,x + 2.50\,y = 1.00 \end{cases} \rightsquigarrow \begin{array}{l} x \approx 3.10\ldots \\ y \approx -2.08\ldots \end{array}$$

vs.

$$\begin{cases} 1.00\,x + 1.01\,y = 1.00 \\ 2.00\,x + 2.10\,y = 1.00 \end{cases} \rightsquigarrow \begin{array}{l} x \approx 13.62\ldots \\ y \approx -12.49\ldots \end{array}$$

$$\begin{cases} 1.00\,x + \overset{1.00}{\cancel{1.01}}\,y = 1.00 \\ 2.00\,x + 2.50\,y = 1.00 \end{cases} \rightsquigarrow \begin{array}{l} x \approx 3.10\ldots \\ y \approx -2.08\ldots \end{array}$$

vs.

$$\begin{cases} 1.00\,x + 1.01\,y = 1.00 \\ 2.00\,x + 2.10\,y = 1.00 \end{cases} \rightsquigarrow \begin{array}{l} x \approx 13.62\ldots \\ y \approx -12.49\ldots \end{array}$$

$$\begin{cases} 1.00\,x + \overset{1.00}{\cancel{1.01}}\,y = 1.00 \\ 2.00\,x + 2.50\,y = 1.00 \end{cases} \rightsquigarrow \begin{aligned} x &\approx \overset{3.00\ldots}{\cancel{3.10\ldots}} \\ y &\approx \underset{-2.00\ldots}{\cancel{-2.08\ldots}} \end{aligned}$$

vs.

$$\begin{cases} 1.00\,x + 1.01\,y = 1.00 \\ 2.00\,x + 2.10\,y = 1.00 \end{cases} \rightsquigarrow \begin{aligned} x &\approx 13.62\ldots \\ y &\approx -12.49\ldots \end{aligned}$$

$$
\begin{cases}
1.00\,x + \overset{1.00}{\cancel{1.01}}\,y = 1.00 \\
2.00\,x + 2.50\,y = 1.00
\end{cases}
\rightsquigarrow
\begin{array}{l}
x \approx \overset{3.00\ldots}{\cancel{3.10\ldots}} \\
y \approx \underset{-2.00\ldots}{-\cancel{2.08\ldots}}
\end{array}
$$

vs.

$$
\begin{cases}
1.00\,x + \overset{1.00}{\cancel{1.01}}\,y = 1.00 \\
2.00\,x + 2.10\,y = 1.00
\end{cases}
\rightsquigarrow
\begin{array}{l}
x \approx 13.62\ldots \\
y \approx -12.49\ldots
\end{array}
$$

$$\begin{cases} 1.00\,x + \overset{1.00}{\cancel{1.01}}\,y = 1.00 \\ 2.00\,x + 2.50\,y = 1.00 \end{cases} \rightsquigarrow \begin{array}{l} x \approx \overset{3.00\ldots}{\cancel{3.10\ldots}} \\ y \approx \underset{-2.00\ldots}{\cancel{-2.08\ldots}} \end{array}$$

vs.

$$\begin{cases} 1.00\,x + \overset{1.00}{\cancel{1.01}}\,y = 1.00 \\ 2.00\,x + 2.10\,y = 1.00 \end{cases} \rightsquigarrow \begin{array}{l} x \approx \overset{10.99\ldots}{\cancel{13.62\ldots}} \\ y \approx \underset{-9.99\ldots}{\cancel{-12.49\ldots}} \end{array}$$

$$\begin{cases} 1.00\,x + \overset{1.00}{\cancel{1.01}}\,y = 1.00 \\ 2.00\,x + 2.50\,y = 1.00 \end{cases} \rightsquigarrow \begin{aligned} x &\approx \overset{3.00\ldots}{\cancel{3.10\ldots}} \\ y &\approx \underset{-2.00\ldots}{\cancel{-2.08\ldots}} \end{aligned}$$

same error

vs.

$$\begin{cases} 1.00\,x + \overset{1.00}{\cancel{1.01}}\,y = 1.00 \\ 2.00\,x + 2.10\,y = 1.00 \end{cases} \rightsquigarrow \begin{aligned} x &\approx \overset{10.99\ldots}{\cancel{13.62\ldots}} \\ y &\approx \underset{-9.99\ldots}{\cancel{-12.49\ldots}} \end{aligned}$$

small

$$\begin{cases} 1.00\,x + \overset{1.00}{\cancel{1.01}}\,y = 1.00 \\ 2.00\,x + 2.50\,y = 1.00 \end{cases} \rightsquigarrow \begin{aligned} x &\approx \overset{3.00\ldots}{\cancel{3.10\ldots}} \\ y &\approx \underset{-2.00\ldots}{-\cancel{2.08\ldots}} \end{aligned}$$

same error

vs.

$$\begin{cases} 1.00\,x + \overset{1.00}{\cancel{1.01}}\,y = 1.00 \\ 2.00\,x + 2.10\,y = 1.00 \end{cases} \rightsquigarrow \begin{aligned} x &\approx \overset{10.99\ldots}{\cancel{13.62\ldots}} \\ y &\approx \underset{-9.99\ldots}{-\cancel{12.49\ldots}} \end{aligned}$$

large

But systems don't always have one solution!

$$\begin{cases} 2.00\,x^2 - 2.10\,xy + 3.10\,y^2 + 7.00 = 0 \\ 7x - 3y + 1 = 0 \end{cases}$$

$$\begin{cases} 2.00x^2 - 2.10xy + 3.10y^2 + 7.00 = 0 \\ 7x - 3y + 1 = 0 \end{cases}$$

zero?

$$\begin{cases} 2.00\,x^2 - 2.10\,xy + 3.10\,y^2 + 7.00 = 0 \\ 7x - 3y + 1 = 0 \end{cases}$$

zero?

One?

$$\begin{cases} 2.00\,x^2 - 2.10\,xy + 3.10\,y^2 + 7.00 = 0 \\ 7x - 3y + 1 = 0 \end{cases}$$

Zero?

One?

Two?

$$\begin{cases} 2.00\,x^2 - 2.10\,xy + 3.10\,y^2 + 7.00 = 0 \\ 7x - 3y + 1 = 0 \end{cases}$$

The more of these, the more solutions

Zero?

One?

Two?

Two ingredients:

Two ingredients:

SENSITIVITY TO ERRORS

Two ingredients:

SENSITIVITY TO ERRORS

NUMBER OF SOLUTIONS

Two ingredients:

SENSITIVITY TO ERRORS

NUMBER OF SOLUTIONS

how do they relate?

My research for the Masses:

My research for the Masses:

The MORE solutions

a system have,

My research for the Masses:

The MORE solutions a system have,

the MORE SENSITIVE TO ERRORS its solutions are!

Thank You

for your Attention!